MATH 261 EXAM 1

1) Find the velocity and acceleration of a particle whose position at time t is given by $\mathbf{r}(t) = (\sin t, \cos t)$. Draw the curve and the vectors.

2) Find $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists. If I define $f(x,y) = \frac{3x^2y}{x^2+y^2}$ for $(x,y) \neq (0,0)$, can f be extended to a continuous function defined on all of $\mathbb{R}^2$? How?

3) For $f(x,y) = \frac{3x^2y}{x^2+y^2}$, compute the first partial derivatives away from zero.

4) For $z = f(x,y) = \frac{3x^2y}{x^2+y^2}$, find the equation of the tangent plane over the point $(1,1)$.

5) At $(1,1)$, in what direction is the function $f(x,y) = \frac{3x^2y}{x^2+y^2}$ increasing fastest?

6) Using the chain rule compute $\frac{d}{dt}(f \circ \mathbf{r})(t)$.

7) Set up the integral giving the volume of the region bounded by the planes $x = 0,\ y = 0,\ z = 0,\ x + y + 2z = 1$.

8) Compute the volume under the surface $z = 1 - x^2 - y^2$ and over the xy-plane.

Bonus: Set up any integral giving the volume of the solid above the cone $z = \sqrt{x^2+y^2}$ and under the sphere $x^2 + y^2 + z^2 = 1$.