

MATH 262 EXAM I

1) Solve:

- $y' + y = 5 \sin(2t)$
- $y' = (e^{-x} - e^x)/(2y - 5)$
- $y'' + 4y' + 3y = e^{-t}$ $y(0) = 2$, $y'(0) = -1$
- $yy'' + (y')^2 = 0$

2) Solve $t^2y'' - 2y = \tan(t)$, $y_1(t) = t^2$. Get y_2 by reduction of order and then solve inhomogeneous problem.

3) Solve $y'' + 4y' + 3y = \delta(t - 3)$, $y(0) = 2$, $y'(0) = -1$.

4) A spring is stretched 10 cm by a 3 N force. A mass of 2 kg is hung from it and attached to a viscous damper exerting a force of 3 N when the velocity of the piston is 5 m/s. If the mass is pulled down from equilibrium 5 cm and given an initial velocity of 10 cm/s downward, determine its position at any time. Is this motion underdamped or overdamped? At what times does the motion reach its first and (if there is a) second extrema? Leaving the mass and spring alone, what value for the damping constant will result in the motion's being critically damped? With the same initial conditions do we get an extremum in the motion or not?