

### IN-CLASS EXERCISE III

First a homework assignment: On p 78 of Rudin do 1, 2, 3, 6, 9.

Here in class we'll do problem 7 after completing last time's example:

*Claim:*  $a_k = 1/k$  is convergent.

*Proof:* For  $\epsilon > 0$  we want  $n, m$  large enough so that  $|\frac{1}{m} - \frac{1}{n}| = |\frac{n-m}{mn}| < \epsilon$ . Simply notice that

$$(1) \quad \left| \frac{n-m}{mn} \right| < \frac{n}{mn} = \frac{1}{m}$$

and this can be made as small as we please by Rudin's theorem 1.20.

Now for Rudin's stuff:

$$7)[\text{Rudin p 78}] \ a_n \geq 0 \ \forall n \Rightarrow (\sum a_n < \infty \Rightarrow \sum \frac{\sqrt{a_n}}{n} < \infty).$$

With theorem 3.11 of Rudin's book we translate the problem to

$$(2) \quad (\forall \epsilon > 0 \ \exists N = N_\epsilon, n \geq N \Rightarrow a_n + a_{n+1} + \dots + a_{n+k} < \epsilon, \ \forall k)$$

$$(3) \quad \Rightarrow (\forall \delta > 0 \ \exists M = M_\delta, n \geq M \Rightarrow \frac{\sqrt{a_n}}{n} + \frac{\sqrt{a_{n+1}}}{n+1} + \dots + \frac{\sqrt{a_{n+k}}}{n+k} < \delta, \ \forall k)$$

Now, **POOF!** if we put

$$(4) \quad \alpha_k = \frac{1}{k} \quad \text{and} \quad \beta_k = \sqrt{a_k}$$

we have

$$(5) \quad \sum_{k=1}^n \alpha_k \beta_k \leq \sqrt{\sum_{k=1}^n \alpha_k^2} \sqrt{\sum_{k=1}^n \beta_k^2}$$

**Why?**

**Complete the proof.**