MATH 396 FINAL EXAM

Notes:

- Please only set up the solutions to the problems. No decimal approximations.
- I have not included any tables–use the usual notation to describe percentiles. E.g. $F_{p,m,n}$ denotes the $100p$th percentile of the F distribution with m numerator degrees of freedom and n denominator degrees of freedom. For normal distribution z_p, for T use $t_{p,n}$, $\chi^2_{p,n}$ for chi-squared, etc.

1) Suppose 100 fair dice are rolled. What is the probability that the sum of the faces showing, S, equals 100? How about $200 \leq S \leq 300$?

2) Recall that in the derivation of the F-distribution we computed the *cdf* $F_{U/V}(t) = P(U \leq tV) = \iint_R f_U f_V$ where $R = \{(u,v) | u \leq tv\} \subset \mathbb{R}^2_+$. Find the corresponding region R for F_{UV}. Assuming U and V are independent random variables from a uniform distribution $f_U(t) = f_V(t) = 1/\theta,\ t \in [0,\theta]$, compute f_{UV} (set up the integrals–*Ans:* $f_{UV}(t) = \frac{\ln(\theta^2/t)}{\theta^2},\ t \in [0,\theta^2]$).

3) Use the method of moments to estimate the parameter α in $f_{UV}(t) = \frac{\ln(\alpha/t)}{\alpha},\ t \in [0,\alpha]$. Is the estimator biased? Compute $\mathrm{Var}(UV)$ and the efficiency of the estimator above. You may find the following integrals useful: $\int_0^\alpha t^n \ln(\alpha/t)dt = \frac{\alpha^{n+1}}{(n+1)^2}$.

4) Find the maximum likelihood estimate for the parameter p in the *pdf*

$$p_X(k) = (1-p)^{k-1}p \quad k = 1, 2, \ldots$$

Is this estimator biased?

5) For data $(y_k)_{k=1}^{10}$ satisfying $\sum y_k = 1100$ and $\sum y_k^2 = 130000$ construct a 90% confidence interval for the mean.

6) Let U and V be independent χ^2 random variables with 7 and 9 degrees of freedom, respectively. Find the probability that $\frac{U/7}{V/9} \in (2.5, 3.3)$.